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SOURCES OF VARIATION IN CATCH AT AGE DATA AND THE OPTIMAL USE OF AGE READING EFFORT

by

J G Pope

and

B Knights

Ministry of Agriculture, Fisheries and Food, Fisheries Laboratory, Lowestoft, Suffolk

ABSTRACT

The applicability of various stratifications to English sampling of the catch at age of North Sea plaice is considered and a method for optimizing the numbers of otoliths read in each strata is suggested. This is a non-linear programming method designed to bring the coefficient of variation of each significant age of fish as close as possible to 10% subject to constraints on the minimum numbers of otoliths to be read per strata and the overall maximum number of otoliths that can be read.

INTRODUCTION

For many fish stocks the major component of the variance of catch at age data is that due to the sampling of otoliths (or other age reading material). Since age reading is a time-consuming occupation, increasing the precision of catch at age data by increasing sample sizes is rarely possible practically. In order therefore to increase precision, stratifications of the catch (length group, quarter of the year landed, port of landing, geographical area of catch, etc) are often introduced in order to sample more homogeneous groupings. The effects of such stratifications have been considered in the past, notably by Gulland (1955), but problems remain: how to chose the 'best' set of strata and how to allocate age reading resources to the various strata? This paper is intended to throw some light on these problems.

SOURCES OF VARIATION

If in an age length key (alk) the proportion of fish aged a in the lth length group is p and the number of fish caught in that group is L then, if A is the number of fish aged a from this group, following Gulland (1955):

$A = p.L$

It follows that the variance of A is

$$var(A) = L^2 . \ var(p) + p^2 . \ var(L)$$

The first term of this equation (which refers to the sampling error of the ageing technique) is in most cases much the larger element and the second term (which refers to the sampling error of length measurements) may often be ignored. This makes for an easier calculation of variance and also concentrates attention on those errors that are expensive to reduce, viz the errors due to the random sampling of otoliths.

Var (p) may be estimated by comparison between alks drawn at random from the same population, or by assuming that p follows the binominal distribution, ie that $var(p) = p. (1-p)/n$ where n is the number of otoliths read in the length group.

Using the latter assumption it is fairly simple to consider the effects of using finer stratifications of the catch at length data and alk data. In practice port of landing, year quarters and subareas are the strata which would seem the most useful. As an example of the effect of such stratification on the variance of the numbers caught at age, consider the decomposition of a primary alk (no suffix), into 2 alks denoted by suffixes 1 and 2. It may be easily demonstrated that there need be no increased variance involved in making such a split since,

$$var(A) = (L_1 + L_2)^2 . p. \ (1 - p)/(n_1 + n_2)$$

$$var(A_1) = L_1^2 . p. \ (1 - p)/n_1$$

$$var(A_2) = L_2^2 . p. \ (1 - p)/n_2$$

when,

$$p = p_1 = p_2$$

and if $L_1/L_2 = n_1/n_2$.

Then it follows that

$$var(A_1 + A_2) = (L_1 + L_2)^2 . \ p(1 - p)/(n_1 + n_2) = var(A).$$

Thus, if the probabilities p_1 and p_2 were the same then, so long as the number of otoliths in the 2 alks were in the same ratio as the numbers of fish of length l in the 2 strata, the variance of the number at age a of length l would be the same whether the primary alk was decomposed into strata or not. If the 2 ratios are not the same, there is a loss of precision involved in the stratification when $p_1 = p_2$ since if,

$L_1/L_2 = b.n_1/n_2$ where b is some constant,

then $var(A_1 + A_2)$ has a minimum at $b = 1$ when it equals var (A).

If p_1 and p_2 differ there can be an advantage in stratification. This can be seen from the following example. If

$L_1 = 2000 \quad L_2 = 1000$

$n_1 = 20 \quad n_2 = 10 \quad p_1 = 0.1 \quad p_2 = 0.2,$

it follows that

$L = 3000 \quad n = 30 \quad p = 0.1333$

and therefore that

var (A) = 34667

while

var $(A_1 + A_2)$ = 34000.

Thus there is a small reduction in overall variance. If the difference were greater, the advantage would be bigger. For example, if we modify the previous example so that

$p_1 = 0.05 \quad p_2 = 0.30,$

then p would still be 0.1333 and var (A) = 34667, but

var $(A_1 + A_2)$ = 30500.

Since there need be no losses and it is possible to make gains using stratifications, it is worth looking at what sorts of differences exist between the more likely strata (areas, quarters, ports). Tables 1, 2 and 3 show the average ages for male plaice length 25-29, 30-34, 35-39 caught at Lowestoft and Grimsby in each quarter in areas 1 + 2 of the North Sea chart shown in Figure 1 (there were insufficient otoliths to consider areas 5 or 6). Tables 1a, 2a, and 3a show the analyses of variance. It may be seen that the most important constituents are ports for the 25-29 length group, quarters for the 30-34 length group and ports for the 35-39 length group. Areas also differed significantly for the 25-29 cm fish and for the 30-34 cm fish. From this work it would appear that all the strata used for sampling North Sea plaice have some value. Since there is no loss in precision from stratification, provided the numbers of otoliths in each strata are chosen correctly, it would seem wise to retain all strata if this is at all possible. Ports and quarters are of course already used in the present North Sea plaice sampling design. Areas however have not previously been used as a stratification for this stock. The use of areas would, however, make the collection of plaice otoliths more complicated and would certainly require considerably more planning of market measurements, but it might be desirable in that it would generate a more highly controlled sampling scheme. One possible economy in the number of strata used for plaice might be to use half years rather than quarter years as the time grouping, since from Tables 1, 2 and 3 it would appear that most differences were between the first and second half years.

OPTIMIZATION OF OTOLITH NUMBERS PER STRATA AND LENGTH GROUP

Having decided to retain the various strata, the question becomes - What is the most efficient distribution of otoliths to each strata and length group? Recalling that according to Gulland (1955) the objective of sampling is to achieve a constant coefficient of variation for all significant ages of fish (CV = 10% say), the problem can be rewritten as follows in a form which allows of an optimal solution. If a function of y is defined so that

$$y = \Sigma\ (\Sigma\ \text{var}\ (A_a)/(\Sigma\ A_a)^2 - 0.01)^2,$$

then clearly the minimum of y will occur when the number of otoliths in each length group and strata are such as to bring all coefficients of variation as close to 10% as is mathematically possible. However, var (A_a) is a function of the number of otoliths in each strata and length group. Consequently, the number of otoliths that are required to reduce the CV of one age of fish to 10% might reduce that of another group to less than 10%. Thus a simultaneous achievement of a 10% CV may not be possible, and in this case the minimum of y could be expected to be a good compromise solution. Of course, other possible objective functions could be developed.

In order to seek only practical solutions, the minimization is made subject to 2 series of constraints. The first of these is that the total number of otoliths read should not exceed some maximum determined by the staff available. This constraint therefore reads:

$\Sigma\ n \leqslant$ some specified total

for all strata.

The second constraint is that a certain minimum number of otoliths be read in each strata and length group, say 5.

Hence $n_l \geqslant 5$

where n_l is the number of otoliths read in the l'th strata/length group. The problem thus becomes an exercise in non-linear mathematical programming which may be solved using standard techniques.

Table 4 shows some results of applying these methods to the 1973 data for male North Sea plaice. It gives the results from the existing sampling scheme (these can be used as a bench mark) and the outcome of various runs of the programmes under differing assumptions. Run 1 was made assuming that up to 1000 otoliths could be read, the same number as were read in areas 1, 2, 3 and 4 in 1973 (areas 5 and 6 are not included in any of these analyses due to insufficient data). The strata used in run 1 were half years, ports and areas. It can be seen that the optimization has resulted in CVs for each age from 2 to 10 that are all lower than the results from the existing system. Run 2 is based on the same strata as run 1 but was limited to 700 otoliths to see if the effort might be reduced and still give results as good as those achieved in the past. Runs 3 and 4 are also based on 700 otoliths and the strata used were quarters and areas for the former and quarters and ports for the latter. It can be seen that runs 2, 3 and 4 all compare favourably with the bench mark particularly in the results for the older ages. There does not seem to be any great difference between these different runs and it would therefore seem that there is no particularly optimal strata set. This is of course in line with the findings of the analyses of variance of Tables 1, 2 and 3.

The distribution of otoliths suggested by the various runs differed in detail but all suggested that effort should be concentrated on Lowestoft rather than Grimsby and that comparatively more effort should be devoted to the 30-34 and 35-39 length groups rather than the less variable 25-29 cm fish and the 40+ fish which were mostly older than 10, the oldest age considered in the optimization.

The results of this section of the report are tentative, being based only on the 1973 catch of male North Sea plaice. It is however likely that for most stocks the use of optimization techniques would achieve some greater precision for an otolith reading effort equivalent to that which is currently used. From the North Sea plaice example it is clear that when the sampling is improved using optimization techniques (Run 1) the results fall somewhat short of the 10% coefficient of variation which would be considered desirable. Thus the use of these techniques would not of itself justify a reduction in effort. In fact, the improvement is rather modest and this suggests that while the optimal solution would define a desirable sampling strategy, suboptimal solutions designed to circumvent practical problems will in many cases only carry a small 'cost' in lost precision. The advantage of performing the optimization is that it enables this 'cost' to be quantified.

REFERENCE

GULLAND, J. A., 1955. Estimation of growth and mortality in commercial fish populations. Fish. Invest. Lond. (2), 18(9), 46 p.

TABLE 1 Average age of fish in each station for ♂ North Sea plaice in the 25-29 cm length group

AREAS	1 + 2		3 + 4	
PORTS / QUARTERS	LT	GY	LT	GY
1st	3.71	4.08	3.54	3.27
2nd	3.74	3.89	2.89	3.57
3rd	3.38	3.50	2.43	3.60
4th	2.89	3.53	2.30	3.60

TABLE 1a Analyses of variance of average age data for 25-29 cm fish

SOURCE	D F	S SGS	M SQ	F
Areas	1	0.77	0.77	7.28
Quarters	3	0.82	0.27	2.58
Ports	1	1.08	1.08	10.17
Residual	10	1.06	0.11	
Total	15	3.74		

TABLE 2 Average age of fish in each stratum for ♂ North Sea plaice in the 30-34 cm length group

AREAS	1 + 2		3 + 4	
QUARTERS / PORTS	LT	GY	LT	GY
1st	4.30	4.83	4.64	4.64
2nd	4.68	4.56	4.00	4.09
3rd	4.03	3.91	3.20	3.90
4th	3.50	3.88	3.14	3.72

TABLE 2a Analyses of variance of average age data for 30-34 cm fish

SOURCE	D F	S SGS	M SQ	F
Areas	1	0.35	0.35	5.65
Quarters	3	2.83	0.94	15.33
Ports	1	0.26	0.26	4.22
Residual	10	0.62	0.06	
Total	15	4.06		

TABLE 3 Average age of fish in each stratum for ♂ North Sea plaice in the 35-39 cm length group

AREAS	1 + 2		3 + 4	
QUARTERS / PORTS	LT	GY	LT	GY
1st	7.30	4.75	6.36	6.92
2nd	7.29	6.83	7.50	6.31
3rd	I O	I O	I O	I O
4th	6.64	6.24	6.86	6.53

TABLE 3a Analyses of variance of average age data for 35-39 cm fish

SOURCE	D F	S SGS	M SQ	F
Areas	1	0.17	0.17	0.39
Quarters	2	0.87	0.43	0.98
Ports	1	1.59	1.59	3.61
Residual	7	3.06	0.44	
Total	11	5.69		

TABLE 4 Coefficient of variation at each age for the current sampling scheme and for various optimizations of the numbers of otoliths in each strata and length group, based on otoliths of ♂ North Sea plaice 1973

Run number	% Coefficient of variation									Total Otoliths read
	Ages									
	2	3	4	5	6	7	8	9	10	
Bench mark*	22	9	8	7	19	47	28	50	22	C1000
Run 1	16	8	7	5	12	30	14	30	12	1000
Run 2	23	10	9	6	14	36	16	34	12	700
Run 3	26	11	7	6	15	27	18	27	14	700
Run 4	23	9	8	7	15	34	15	31	11	700

*Bench mark indicates the precision achieved by the current sampling scheme

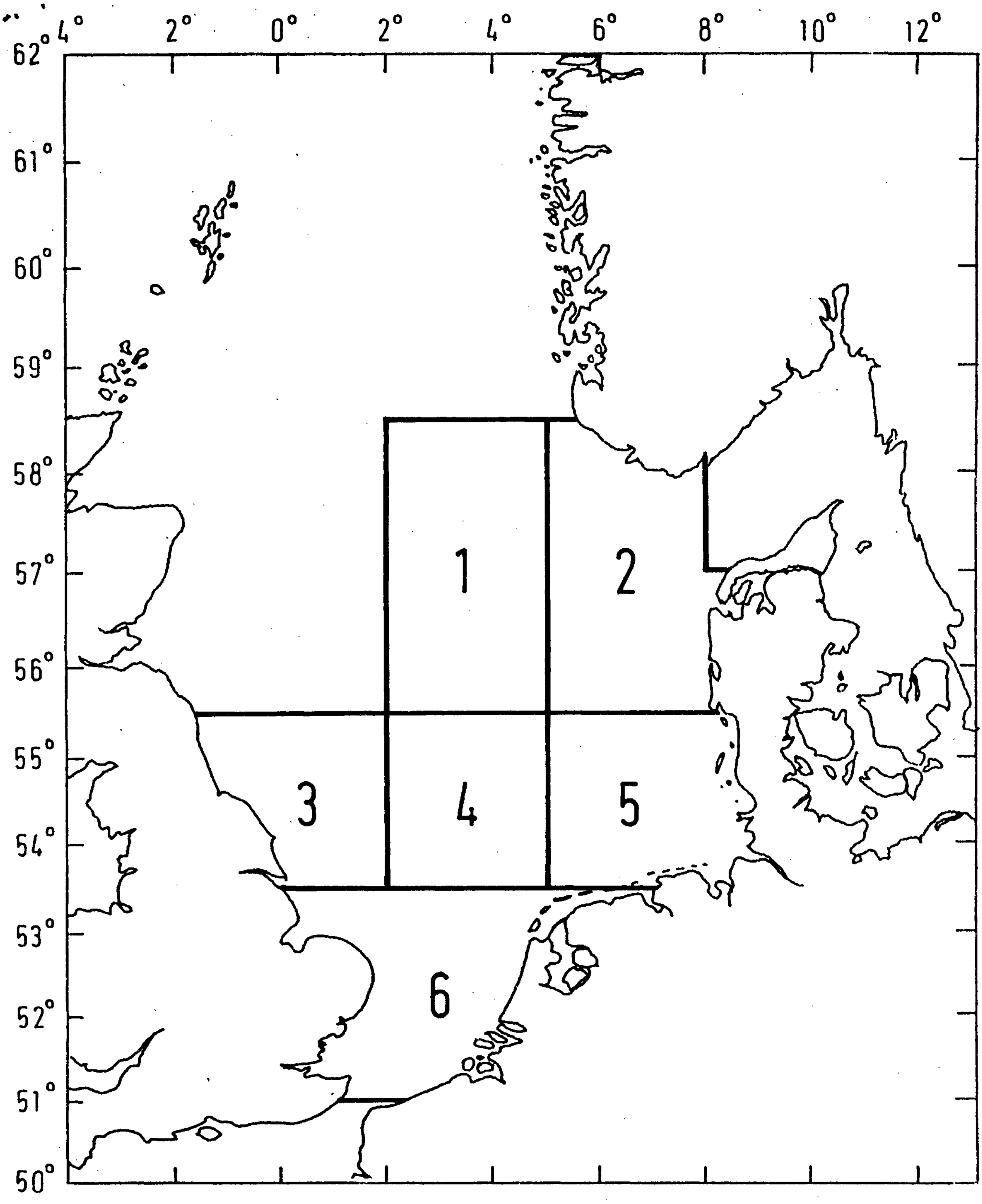

Figure 1 Chart showing sub-areas of the North Sea used to stratify English plaice sampling